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A RESPONSE TO PIGNALOSA & TRABUCCHI AND PERRI & ORO

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In Part II of the *Philosophical Investigations*, Wittgenstein, in his detailed discussion on the psychological problem presented by the 'duck-rabbit' image derived from Jastrow's *Facts and Fable in Psychology*, writes:

> Imagine a physiological explanation of the experience. Let it be this: When we look at the figure, our eyes scan it repeatedly, always following a particular path. The path corresponds to a particular pattern of oscillation of the eyeballs in the act of looking. It is possible to jump from one such pattern to another and for the two to alternate. (Aspects A) Certain patterns of movements are physiologically impossible; hence, for example, I cannot see the schematic cube as two interpenetrating prisms. And so on. Let this be the explanation. –'Yes, that shews it is a kind of *seeing*.' –You have now introduced a new, a physiological, criterion for seeing. And this can screen the old problem from view, but not solve it. –The purpose of this paragraph however, was to bring before our view what happens when a physiological explanation is offered. The psychological concept hangs out of reach of this explanation. And this makes the nature of the problem clearer.
>
> (Wittgenstein 1953, 212e)

This is the core of my argument. Sraffa's *physical* explanation of the rate of profits puts Adam Smith's *psychological* concept of the rate of profits out of reach and this makes the nature of the problem clearer. My critics and Garegnani's followers, however, insist that Adam Smith's psychological explanation must be inserted in Sraffa's physical explanation, which is an impossibility. Before going any further, let me briefly reiterate what I consider to be Sraffa's *physical* explanation of the rate of profits.

Let us suppose that a subsistence system or a system that produces no surplus is given by:

$$280 \text{ qr. wheat} + 12 \text{ t. iron} \longrightarrow 400 \text{ qr. wheat}$$
$$120 \text{ qr. wheat} + 8 \text{ t. iron} \longrightarrow 20 \text{ t. iron}.$$

Which, in terms of its values can be written as:

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$$\begin{aligned} 280\,p_w + 12\,p_i &= 400\,p_w \\ 120\,p_w + 8\,p_i &= 20\,p_i \\ \hline 400\,p_w + 20\,p_i &= 400\,p_w + 20\,p_i \end{aligned} \qquad \text{(i)}$$

In this case, without any additional information one could derive the unique exchange ratio between the two commodities as 10 qr. wheat for 1 ton of iron that would ensure that this system would reproduce itself at the same scale. Now rescale this system to a stage where its reproduction at the same scale is no longer possible. In other words, the system is not in equilibrium.

280 qr. wheat + 12 t. iron → 400 qr. wheat
240 qr. wheat + 16 t. iron → 40 t. iron.

Now, suppose that rates of profits emerge in two industries but given that it is a no-surplus system, its total profits equals zero. In other words, if one rate of profits is positive then the other must be negative.

$$\begin{aligned} (280\,p_w + 12\,p_i)\,(1 + r^w) &= 400\,p_w \\ (240\,p_w + 16\,p_i)\,(1 + r^i) &= 40\,p_i \\ \hline (520\,p_w + 28\,p_i)\,(1 + 0) &= 400\,p_w + 40\,p_i \end{aligned} \qquad \text{(ii)}$$

Obviously this system is not producing enough of wheat to reproduce itself at the same scale. The reader can verify from the aggregate of the two equations that the exchange ratio between iron and wheat in this case also remains 10 units of wheat for 1 unit of iron.[1] Furthermore, by plugging the value of p_w in the two equations we can verify that both $r^w = r^i = 0$. Thus both prices and the rate of profits in this system are determined by the interconnected structure of the system of production without having to introduce any more information from outside. Now, the question is: does this property of an interconnected system remain valid for a surplus producing system of production as well? My argument is that Sraffa (1960) goes on to show that it indeed is valid for an interconnected surplus producing system of production as well – this is the reason why Sraffa's book begins with a short chapter on no-surplus economy. The argument runs as follows:

[1] In his response to Roy Harrod's (1961) review of his book, where Sir Roy had suggested that Sraffa was implicitly assuming equilibrium of demand and supply in his price equations, Sraffa (1962) responded that '(…[even when] the system ceased to be in self-replacing state) the exchange ratio would remain the same but the ratio between the excess productions of the two commodities would be changed, so that the two would no longer be equal' (477). In other words, the prices are determined by the methods of production and not the condition of equilibrium of demand and supply. As we shall see, this proposition for Sraffa had a general validity and was not only applicable to a 'subsistence system'.

Let us suppose we have a three-good economy given by:

90 ton Iron + 120 ton Coal + 60 qr. Wheat → 180 ton Iron
50 ton Iron + 125 ton Coal + 150 qr. Wheat → 450 ton Coal
40 ton Iron + 40 ton Coal + 200 qr. Wheat → 480 qr. Wheat

180 t. Iron + 285 t. Coal + 410 qr. Wheat → 180 t. Iron + 450 t. Coal + 480 qr. Wheat

In value terms we could express this economy by a set of equations, given by:

$$\begin{array}{l}(90\,p_i + 120\,p_c + 60\,p_w)\,(1 + r^i) = 180\,p_i \\ (50\,p_i + 125\,p_c + 150\,p_w)\,(1 + r^c) = 450\,p_c \\ (40\,p_i + 40\,p_c + 200\,p_w)\,(1 + r^w) = 480\,p_w \\ \hline (180\,p_i + 285\,p_c + 410\,p_w)\,(1 + \mathrm{R}) = 180\,p_i + 450\,p_c + 480\,p_w\end{array} \qquad \text{(iii)}$$

Our problem is to solve for r's and p_js (call it a set of **P**). The challenge is to see if there is enough information in this description of the interconnected system of production or a system of basic goods[1] that will allow us to find out the values of the unknowns. In other words, the value of no unknown can be taken as given from outside. There may exist no solution, unique solution or multiple solutions.

Restrictions imposed by economic reasoning on the system (iii): (1) all prices must be strictly positive and (2), all r's must be semi-positive.

Let us assume multiple solutions exist. That will generate multiple R's.

Now we rescale this system by multiplying the first equation by 4/3 and the second equation by 4/5:

$$\begin{array}{l}(120\,p_i + 160\,p_c + 80\,p_w)\,(1 + r^i) = 240\,p_i \\ (40\,p_i + 100\,p_c + 120\,p_w)\,(1 + r^c) = 360\,p_c \\ (40\,p_i + 40\,p_c + 200\,p_w)\,(1 + r^w) = 480\,p_w \\ \hline (200\,p_i + 300\,p_c + 400\,p_w)\,(1 + \mathrm{R}^\star) = 240\,p_i + 360\,p_c + 480\,p_w\end{array} \qquad \text{(iv)}$$

We know that multiplying any equation by a constant does not add to or subtract from the information that already exists in the original equation. System (iv) is what Sraffa calls the Standard system. The average rate of profits of the equation system (iv), *i.e.*, $\mathrm{R}^\star$, is equal to 1/5 or 20%. This is independent of the values of *p's* since (40 iron + 60 coal + 80 wheat)/(200 iron + 300 coal + 400 wheat) is a ratio of heterogeneous goods arranged in equal proportion and therefore, its ratio is math-

[1] Sraffa defines a basic good as a good that directly or indirectly enters as an input in the production of all goods.

ematically well defined and equals 1/5. It is now revealed that in the system of equation (iv), which has the same information as the system of equation (iii), the average rate of profits turns out to be a physical property of the system – it is an algebraic result derived by simply aggregating the original set of equations and not a statistical result based on whether the classical 'market prices' or the 'natural prices' prevail. As we shall see below, Sraffa's contention is that since the equation systems (iii) and (iv) are mathematically equivalent systems, the same algebraic property must hold for the equation system (iii), *i.e.*, R must be equal to R*, which is equal to 20% in this case. Now the question is: does this imply that all the *r*'s in the equation system (iii) must be uniform? That depends on whether equation system (iii) has unique R or several R's. Sraffa (1960), however, has also proved that for every system of basic goods there exists one and only one Standard system, which rules out multiple R's for the system (Sraffa 1960, for a complete proof also see Lippi 2008). Now that the equation system (iv) remains the Standard system for all the rescaled systems derived from the equation system (iii), it follows that their average rate of profits must also be 20%. But this is guaranteed, if and only if, all the *r*'s in the equation system (iii) are uniform or equal to 20%.[1] This is why Sraffa could state, without any qualification, that 'the rate of profits … *must* be uniform for all industries …' (Sraffa 1960, 6, emphasis added). This result is independent of any notion of equilibrium of demand and supply as it does not take into consideration any information from the side of the final demands of the commodities produced – it is simply an accounting of what is produced and how it is appropriated.

We have now shown that any observed system of basic goods (*i.e.*, interconnected industries) has enough internal structural constraint that it does not allow prices and the industrial rates of profits any freedom – the system's average rate of profits is determined by its physical properties alone and that constrains the industrial rates of profits and prices in such a way that they can have only a unique solution. This leaves all the psychological explanations of the rates of profit and prices out of reach since in the psychological explanation prices are determined by the interaction of demand and supply, which in turn determines individual industrial rates of profits. In his psychological explanation of the rates of profits, Adam Smith writes:

> When the quantity of any commodity which is brought to market falls short of the effectual demand, all those who are willing to pay the whole value of the rent, wages, and profit, which must be paid in order to bring it thither, cannot be supplied with

[1] For a general and formal statement of the above description, see Appendix, written jointly with Sanjay Reddy.

the quantity which they want. Rather than want it altogether, some of them will be willing to give more. A competition will immediately begin among them, and the market price will rise more or less above the natural price, according as either the greatness of the deficiency, or the wealth and wanton luxury of the competitors, happen to animate more or less the eagerness of the competition … [T]he quantity brought to market should at any time fall short of the effectual demand, some of the component parts of its price must rise above their natural rate.

(Smith 1976 [1776], 73-75)

The reader would have noticed the dissonance between Smith's psychological description of the determination of the market price of *a* commodity and its impact on the rate of profits in that industry and Sraffa's interdependent industries, where both inputs and outputs are accounted for by the same price. Because in Sraffa's case, it is simply not possible to speak of a rise or fall in the price of *a* commodity since a change in the price of one commodity would at the same time must affect the prices of all commodities. Thus prices must be spoken in terms of a set, which must be determined simultaneously. In other words, in Sraffa's system '*a* commodity' is not produced – the system produces *n* commodities altogether and if even one commodity is deleted from the system then it reduces to zero commodity producing system. The nature of its production as production of a 'composite commodity' is revealed by its associated Standard system and the Standard commodity.

So, is Adam Smith simply wrong or is he coming from a conceptual framework that is not shared by Sraffa? In Chapter V of the *Wealth of Nations*, on 'of the real and nominal price of commodities' Adam Smith clearly states his position in this matter:

Labour was the first price, the original purchase money that was paid for all things. It was not by gold or by silver, but by labour, that all the wealth of the world was originally purchased; …

(Smith 1976 [1776], 48)

What Adam Smith is alluding to here is the concept of complete vertical integration of an industry, *i.e.*, the technique of production of a commodity can be reduced to a long series of labouring activity culminating in the primordial state where labour works against Nature unaided by any produced means of production. If all the industries could be completely vertically integrated in this manner then all such industries would be independent of each other as they would themselves produce all their means of production and therefore, a change in the price of one commodity will have no effect on the cost of production of any other commodity.[1]

[1] It should be noted that Smith ignores the impact of income changes, due to change in the price of *a* commodity, on the demand for other commodities and thus their prices, which makes his gravitation mechanism simpler but, of course, not robust. Also see Dupertuis and Sinha (2009a) for a critique of the classical notion of the centre of gravitation.

In this case there is no basic good in the system, only final commodities exchange, prices of each commodity is determined by the demand and supply conditions. The interactions between industries take place due to another psychological assumption that capitalists seek to maximize their rate of profits and therefore, resources flow into the industries with higher rates of profits from industries with lower rates of profits bringing the industrial rates of profits to parity once again through the workings of the same demand and supply price mechanism.

Sraffa rejects this reductionist framework. As early as 1928-1931 period, Sraffa had realized that if industries are interconnected, *i.e.*, if industries were required to buy their commodity inputs (means of production) from each other, then, no matter how far back one goes in the production chain, one will always be left with a *commodity residue*. In other words, the road to the primordial or the originary state of production was logically blocked and hence the reductionist framework had to be given up. This realization was momentous in the development of Sraffa's ideas. Sraffa could immediately see that it had serious consequences for economic theory – within the reductionist framework a zero wage to labour must imply an infinite rate of profits; however, in Sraffa's structuralist framework, a zero wage must be associated with a finite maximum rate of profits – this, *prima facie*, changes the theory of the relationship between wages and profit. Later Sraffa (1960) credits Marx for being the first to have discovered the idea of *commodity residue*.

> The notion of a Maximum rate of profits corresponding to a zero wage has been suggested by Marx, directly through an incidental allusion to the possibility of a fall in the rate of profits 'even if the workers could live on air'; but more generally owing to his emphatic rejection of the claim of Adam Smith and others after him that the price of every commodity 'either immediately or ultimately' resolves itself entirely (that is, to say, without leaving any commodity residue) into wage, profit and rent – a claim which necessarily presupposed the existence of 'ultimate' commodities produced by pure labour without means of production except land, and which therefore was incompatible with a fixed limit to the rise in the rate of profits.
>
> (Sraffa 1960, Appendix D, 94)

The next crucial development in Sraffa's ideas takes place in 1942, soon after he got back to work on his book after a decade long break since 1931-1932, when he hypothesizes that the maximum rate of profits associated with zero wages must be a constant with respect to changes in prices due to changes in the rate of profits – this is conceived in the context of an analysis of the relationship between the rate of profits and wages in the backdrop of the discovery of the finite maximum rate of profits that an interconnected system of industries must have. On the condition that industrial rates of profits must be equal, Ricardo (1821) had already shown that in general a change in the rate of profits or

wages would bring about changes in all prices. Sraffa's challenge was to show that any such changes in prices will not affect the value of the maximum rate of profits – in other words, the maximum rate of profits is a physical property of the system and thus cannot be affected by changes in prices.

> What is demanded of the model is that it should show a constant (constant with respect to variations of r) ratio between quantity of capital & quantity of product. If this can be constructed, and proved to be general, a number of important 'consequences' follow.
>
> (Sraffa n.d., D3/12/16: 14, August 1942)

Sraffa's attempt to prove his 'Hypothesis' led him to the discovery of the Standard system and the Standard commodity, which proved that indeed when the Standard commodity (*i.e.*, the average commodity of the observed system) is used as the *numéraire* to measure prices and the wages then it can be shown that the maximum rate of profits, R, remains fixed with respect to changes in prices due to changes in the rate of profits *r*, which gives rise to the most fundamental structural relationship of any given interconnected system of production between the measure of its surplus produced, represented by R, and the two distributional components of it given by *r* and $\boldsymbol{w}$, *i.e.*, $r = \mathrm{R}\,(1 - \boldsymbol{w})$. As a matter of fact, $r = \mathrm{R}^{\star}\,(1 - \boldsymbol{w})$ can be observed as a physical relationship independent of prices, where *r* throughout is measured as the average rate of profits of the Standard system – it is from this particular observation that Sraffa concludes that the average rate of profits of the actual system must also be equal to it in terms of their aggregate values, since the two systems must have the same mathematical properties:

> Such a relation is of interest only if it can be shown that its application is not limited to the imaginary Standard system but is capable of being extended to the actual economic system of observation … But the actual system consists of the same basic equations as the Standard system, only in different proportions; so that, once the wage is given, the rate of profits is determined for both systems regardless of the proportions of the equations in either of them. Particular proportions, such as the Standard ones, may give transparency to a system and render visible what was hidden, but they cannot alter its mathematical properties … The same rate of profits, which in the Standard system is obtained as a ratio between *quantities* of commodities, will in the actual system result from the ratio of aggregate *values*.
>
> (Sraffa 1960, 22-23, emphasis in original)

To confirm this in Sraffa's original examples with wages, we add the Standard commodity as the *numéraire*:

$$\begin{aligned}(90\,p_i + 120\,p_c + 60\,p_w)\,(1 + r) + 3/16\,\boldsymbol{w} &= 180\,p_i \\ (50\,p_i + 125\,p_c + 150\,p_w)\,(1 + r) + 5/16\,\boldsymbol{w} &= 450\,p_c \end{aligned} \qquad \text{(v)}$$

$$(40\ p_i + 40\ p_c + 200\ p_w)\ (1 + r) + 8/16\ \boldsymbol{w} = 480\ p_w$$
$$(40\ p_i + 60\ p_c + 80\ p_w) = 1$$

We find that for $r = 1/5$, we get $p_i = 11/920$, $p_c = 1/230$, $p_w = 3/920$ and $\boldsymbol{w} = 0$; for $r = 1/10$, we get $p_i = 24911/2218720$, $p_c = 4857/1109360$, $p_w = 7993/2218720$ and $\boldsymbol{w} = 1/2$; for $r = 1/20$, we get $p_i = 10979/1008480$, $p_c = 2213/504240$, p_w 3797/1008480, $\boldsymbol{w} = 3/4$ and also for $r = 0$, we get $\boldsymbol{w} = 1$. This proves that r and $\boldsymbol{w}$ are linearly related with R remaining constant at 1/5, *i.e.*, $R = r/(1 - \boldsymbol{w}) = 1/5$ for all values of r.[1]

This clarifies the nature of the problem. In the psychological framework of the price theory, all industries are independent of each other, the price of a commodity is determined by the psychology of the demanders given the quantity supplied, which in turn determines income distribution in the economy, which in turn determines the allocation of given resources in the various industries in the economy till the economy reaches a state of equilibrium where all agents are psychologically satisfied in the sense that no one wants to change the given situation. On the other hand, in the physical framework of Sraffa's price theory, an economy is conceived as a singular animal of interconnected industries. Commodities exchange in the market because, for the reproduction or the survival of the animal, all industries need to buy from and sell their outputs to each other since their outputs are directly or indirectly inputs in the production of all commodities. In this case, if industries produce no surplus outputs then prices of all commodities are uniquely determined by the physical structure of their inputs and outputs. If the system produces a surplus then the measure of the surplus in terms of its maximum rate of profits can be determined by the physical data of the inputs and outputs and prices always must be such that it accounts for that physical measure in nominal terms, which in turn shows that any arbitrary distribution of the surplus in terms of the rate of profits and wages, which is determined independently of prices, must constrain the prices to be such that those distributional measures are accounted for in nominal terms – the significance of the Standard commodity as the Standard of measure for prices and wages is that, being the average commodity of the system, it is not affected by price changes due to changes in the rate of profits and thus the rate of profits measured in the production of this commodity always remains the average rate of profits of the system. In this case the role of prices reduces to homogenizing heterogeneous physical data to a homogeneous scale

[1] This example can be generalized as a proof of Sraffa's 'Hypothesis'. I thank Yoann Verger for those computations.

– neither human psychology plays any role in its determination nor does it play any role in determining human action. If there is no basic good in the system then Adam Smith's market mechanics prevails and Sraffa does not get to play; but if there is at least one basic good in the system then Sraffa prevails – they are mutually exclusive.

This brings us to the conceptual difference between our notion of 'algebraic average rate of profits' and the 'statistical average rate of profits'. If industries are independent of each other, as in Adam Smith's case, then there will be no Standard system and a Standard commodity associated with them. In this case the psychological determination of market prices will give rise to a statistical average rate of profits of the system and any change in demand conditions would change the average rate of profits of a given system of production. As no Standard system exists in this case, all the rescaling of the equation system (iii) would simply generate average rates of profits of the rescaled systems as unknowns R', R'', etc. without ever giving its algebraic value independently of the knowledge of prices and therefore, there cannot be any presumption about the equality of these averages without the knowledge of prices. But in Sraffa's interconnected industries of basic goods there always exist a unique Standard system, which reveals that the interconnected nature of the system constrains the system to a unique average rate of profits – 'the rate of profits is embedded "in the things" and no manipulation of prices could ever affect it. [There could be no more tangible evidence (convincing proof) of the rate of profits [being, as] a non-price phenomenon (effect)]' (Sraffa n.d., D3/12/53: 32, 1955, all large brackets and parentheses are in the original). In this case, prices cannot be taken to be determined by the psychological factors in the market – it is rather the physical average rate of profits that constrains the prices to one and only one set.

As a matter of fact, in 1943, during the development of the concepts of the Standard system and the Standard commodity, Besicovitch suggested to Sraffa that there could be several Standard systems associated with a given system. That caused serious anguish to Sraffa, whose one of the musings at this news is interesting to note:

α) At r max the ratio (Old Hyp.) of Cap. to Rev. in every Actual System is the same as in Standard System
β) But in Stand. Syst. the ratio is the same for Values and any possible prices. Does it follow that the same is true in any Actual System? [No].
γ) The α holds because at r max the ratio is the same in each equation, and therefore in aggregate. In β it holds for the aggregate of Standard System, not necessarily for every equation in it.

Yet the aggregate is not a statistical result, but an algebraic one. (D3/12/36: 79)

In other words, Sraffa's point is that since the aggregate of the Standard system is an algebraic manipulation of the system of equations and the fact that it reveals an average rate of profits of the system independently of prices, this information must be embedded in the original equation system itself and therefore is not a statistical result based on what prices prevail in the market. And since it is clear that there cannot be more than one average rate of profits in the Standard system then how could there be more than one average rate of profits in the actual system as the Standard system is derived by only an algebraic manipulation of the actual system. The idea that the aggregate of the Standard system represents the average properties of the observed system is yet again asserted by Sraffa soon after the publication of the book:

> There are besides, many possible applications [of the Standard commodity], which I have not mentioned in the book {Sraffa 1960}, in problems discussed by Marx. Take, *e.g.* the determination of a general rate of profits, from the rate of surplus value: Marx takes an average of the rates of profits obtained in the production of the different commodities on the basis of "values", and gets, as he acknowledges, an approximately correct result. An exact result could however be obtained by taking, instead of a simple average, a weighted average: & it can be shown that the appropriate weights can be derived directly from the proportions in which the comm.{oddities} enter the "St{andard} com{modity}".
>
> (Sraffa's draft response (written on 12.2.1961) to Eaton's[1] review of his book, Sraffa n.d., D3/12/111: 132, underlining and large bracket in the original)

My critics insist that Sraffa's condition of the equal rate of profits in his system of equations necessarily relies on Adam Smith's psychological explanation of the average rate of profits – it so happens, they argue, that Sraffa, without ever explicitly stating, tacitly assumed his system to be in Adam Smith's centre of gravitation or a state of repose, which assumes an equal rate of industrial profits. They, however, never ask the question: since the average rate of profits in the Standard system is without a doubt a *physical property of the system*, how could its nature turn into a *psychological property* just by rescaling the equations?

Furthermore, in several of my publications (*e.g.*, Sinha 2012, 2018, etc.) I have pointed out evidence from Sraffa's writings, including his published book (Sraffa 1960), that clearly point to the fact that Sraffa

[1] John Eaton's (real name: Bodington) review was published in Italian in the journal *Società* and brought to Sraffa's notice by Maurice Dobb. See Bellino (2006) for the English translation of Eaton's review and the entire draft response of Sraffa.

had rejected the assumption of the classical centre of gravitation. For example: (i) Adam Smith and the classical economists who came after him, including Garegnani, firmly believed that it could only be by a fluke that one could find an actual observed system in the centre of gravitation – the actual system is supposed to fluctuate around it – then why Sraffa chose to refer to his system of equations in the above cited quotation (Sraffa 1960, 22-23) as 'the actual economic system of observation' if he had assumed his system to be in the classical centre of gravitation? (ii) Why would he open the Preface to his book by the clarion declaration, 'Anyone accustomed to think in terms of the equilibrium of demand and supply may be inclined, on reading these pages, to suppose that the argument rests on a tacit assumption of constant returns in all industries … In fact, however, no such assumption is made' (Sraffa 1960, v), when the classical centre of gravitation is defined by the equilibrium of effectual demand and quantity supplied and the mechanism of gravitation of 'market prices' to the 'natural prices' works on the tacit assumption of constant returns? (iii) How can an 'observed or actual system that lies outside of the centre of gravitation be adjusted to a system in the centre of gravitation without any assumption about returns to scale? (iv) Why would Sraffa begin with a declaration, 'This is no system of equilibrium' in a draft of the Preface to his book written in 1957 (see Sinha 2016 for details) when his system was built on the assumption of equilibrium? (v) In 1968, in his response to a German student Soltwedel who had characterised Sraffa's equations to be in equilibrium, why would Sraffa write, 'I must say frankly that you have gone astray the moment you speak of "equilibrium"…' (Sinha 2016) if he himself had implicitly assumed equilibrium for his system of equations? (vi) Why would Sraffa repeatedly refer to his system of equations as a 'photograph' when it so happens that the centre of gravitation is an ideal condition and not a real one and therefore the metaphor of a photograph would be most inapt for it? (vii) Why would Sraffa (1960) claim that his analysis remains valid even for non-self-replacing systems, where some inputs in the system turn out to be larger than their outputs, as such a system is typically associated with dynamic non-equilibrium situations when some industries are going through technical changes or new products are replacing old ones (see Sinha 2022a)? (viii) Why would he spend a full appendix (Appendix B) in his book on the problem of a non-basic good 'beans' that could not generate as high a rate of profits as given by the basic good system when it is clear that in a classical centre of gravitation no industry such as 'beans' could logically exist? (ix) Why would he care to mention explicitly all the assumptions he makes, even minor ones, but keep such a crucial assumption of his theory implicit? (x) Why would he draft two epigraphs for his book

stating, 'A dividend could be declared before knowing what is the price of the company's product' and that the Standard system 'provides tangible evidence of the rate of profits as a non-price phenomenon' if his position was that the rate of profits and prices are determined *simultaneously*? and (xi) Why does he not recognize Adam Smith for such a central concept of his book in Appendix D, where he explicitly acknowledges all others for the crucial ideas of his book? I could go on but this should suffice for now.

To put it succinctly, the basic difference between Garegnani led interpretation of Sraffa and mine is that Garegnani and his followers argue that prices are not determined by the equations of Sraffa. There is a separate realm in which prices are determined and therefore, there is a separate theory that explains it. For them, Sraffa's production equations can only provide the quantity supplied of the commodities in the market. The prices of those commodities, however, are determined by the interactions of the demands with the quantities supplied in the market. If the supplied quantities turn out to be more or less than what is considered to be the 'effectual demands' or the 'equilibrium' in the market, which is precisely defined by the condition that prices must be such that all industries receive equal rate of profits, then the market forces of demand and supply (*i.e.*, an intra-industry force) will generate a set of 'market prices'. And when those 'market prices' are applied to Sraffa's equations then they generate unequal rates of industrial profits; and if we care to aggregate all the equations then we can also compute a 'statistical average rate of profits', which, however, has no analytical significance. Now, given those 'market prices' associated with unequal rates of profits another force comes into play in the space where prices are determined – it is the inter-industry competitive force. Capital and labour begin to move from low profit industries to high profit industries affecting the quantities of supplies in the market and thus influencing the 'market prices' by influencing the first force, *i.e.*, the intra-industry demand and supply force. It is assumed that quantity adjustments do not affect the cost conditions of the industries and therefore their production equations, except for rescaling factors. Now these two forces together drive the 'market prices' in a direction that eventually brings them all to a set which generates equal industrial rate of profits for all. This set of prices is called the equilibrium or the 'natural prices'. It is claimed that since Sraffa imposes a uniform rate of profits on his system of equations, he, from the very beginning, must have assumed that the quantities produced in his equations were exactly equal to the 'effectual demands' in the market to ensure that all the industrial rates of profits must be uniform. In this story it is never, however, explained: where from Sraffa could get the data on the so-called 'effectual demands' of all

the commodities and how could he know how to adjust inputs of the observed technique to those outputs equal to the effectual demands without the knowledge of returns to scale? – a knowledge Sraffa emphatically denied having or assuming.

My interpretation, on the other hand, is rather simple and straightforward and fits in well with Sraffa's utterances. I claim that Sraffa notes down the inputs and outputs of all the industries after the 'harvest' or a production cycle. He identifies the basic goods and deletes all the non-basic goods from his data and also uses a theory of rent similar to Ricardo's to remove all the non-reproducible inputs from his data. Then he constructs a system of equations in value terms of all basic goods. He shows that such a system of interconnected industries can always be algebraically manipulated to form a unique Standard system where one can directly observe, *i.e.*, in physical terms, that the system has a finite maximum rate of profits and an average rate of profits, given wages in terms of the Standard commodity. Further on, one can observe, in physical terms, that the maximum rate of profits remains constant when the total net product is distributed between capitalists and workers as an average rate of profits on capital advanced and wages in terms of the Standard commodity per unit of homogeneous labour. Given that the Actual system and the Standard system are mathematically equivalent systems, the mathematical properties of the Standard system must also hold for the Actual system. One of the consequences of this is that the average rate of profits of the Standard system must hold for every industry in the Actual system, which is associated with a unique set of all positive prices and wages measured in terms of the Standard commodity. This explanation of the prices or the so-called 'natural prices' does not require any information from the demand side of the market or the knowledge of returns to scale from the supply side of the market and hence has no truck with the notion of 'equilibrium' of demand and supply.

Where Do Pignalosa and Trabucchi Go Wrong?

Pignalosa and Trabucchi show their lack of understanding of the significance of the two most crucial aspects of Sraffa's analysis; namely, the concept of the maximum rate of profits associated with zero wages (*i.e.*, the commodity residue) and his 'Hypothesis' that the maximum rate of profits remains constant with respect to changes in the values of the rate of profits, given the condition that the industrial rates of profits must be uniform. This is why early on in their paper they quite nonchalantly declare 'not to follow Sraffa (and Sinha) in assuming that the various quantities of iron, coal and wheat appearing on the left-hand side

of the above table are used only as means of production. We shall assume instead that those quantities include both such means of production and the subsistence wage that is paid to the workers at the beginning of the yearly production cycle' (P & T, p. 160). And later on they go on to add, 'In this case, the average rate of profit of the Standard system can be determined independently of prices only (a) by assuming zero wages (so that the average and the maximum rates of profits coincide), or (b) by using the inverse (and linear) relation that, as Sraffa has shown, can be derived in the Standard system between the rate of profit and the wage rate. However, … the assumption to take zero wages is clearly untenable, while, in order to take wages as the independent variable in the inverse relation holding in the Standard system between the rate of profit and the wage rate, wages must be assumed to consist of Standard commodity; and, as we have seen, this is a composite commodity consisting of consumption goods (in proportion which do not directly reflect workers' consumption habits or choices) and of means of production' (P & T, p. 163, fn. 1).

They seem to not know why Sraffa chose to take wages out of the input matrix of the system as well as not consider it a part of capital advanced. As far as the subsistence wage is concerned, that would not cause much problem if it could be conceived as feed for the horses or fuel for the furnace. But Sraffa's 'Hypothesis' required an analysis of variation of wages from 0 to 1, where 1 happens to be the measure of both actual net output as well as the Standard net output in terms of the Standard commodity. Thus his conceptual framework required him to assume that wages would also contain surplus over and above the minimum subsistence. If subsistence wage could be strictly well defined in physical terms and would not get affected by the addition of surplus to it then Sraffa could easily take only the surplus part explicitly on the side of wages in his equations and define zero wage as zero surplus over and above subsistence. But he decides not to do so because it is obvious that if a worker could afford steak then his requirement for corn may not remain the same. But more importantly, one should ask, why Sraffa breaks from the Classical (and Marxian) tradition of taking wages as part of capital advanced? In the book (Sraffa 1960, 10) he simply states, without any explanation, that 'We shall also hereafter assume that the wage is paid *post factum* … , thus abandoning the classical economists' idea of a wage "advanced" from capital.' But after analysing the significance of the relation between the Actual system and the Standard system and also his unpublished notes on his 'Hypothesis', the reason becomes clear. The 'Hypothesis' requires a linear relationship between $\boldsymbol{w}$ and r [*i.e.*, $r = \mathrm{R}(1 - \boldsymbol{w})$, where R is constant with respect to changes in r] but if wages are taken as part of capital then the relationship becomes

non-linear – as Sraffa wrote during the development of these ideas in 1943-1944 period, 'To obtain the linear relation we must assume wages paid out of the product, not advanced' (Sraffa n.d., D3/12/36: 67-69). The relationship of the maximum rate of profits (*i.e.*, the commodity residue) with the Standard system is so close in his mind that in the same note quoted above, he wrote, 'But we need not go to the trouble of actually working it out {*i.e.*, the Standard system}. The knowledge of R and of the quantity of labour annually employed by society, suffice to construct it.' and further on in another note he wrote, 'Thus (given an Actual System) finding R and finding the Standard System are the same thing (R plus the labour force are the St. System)' (Sraffa, n.d., D3/12/36, 1 February 1944, all parentheses in original).

But if we follow P & T's reasoning then we must conclude that the core of Sraffa's book, including the relation given by $r = R(1 - \boldsymbol{w})$ is either irrelevant or meaningless – this is not a facetious remark. It is a conclusion that one must arrive at if one approaches Sraffa's work from the perspective of the classical centre of gravitation. Garegnani and his followers never understood the significance of the Standard system and the Standard commodity for Sraffa's own analysis – it was relegated to his idiosyncrasy of solving 'Ricardo's problem', *i.e.*, of keeping the size of the net output constant as its distribution between wages and profits changes, in the middle of his book; even when Sraffa himself had clearly stated in para 32, page 23 (Sraffa 1960), that the Standard commodity does not solve 'Ricardo's problem' and in Appendix D, page 93, he had explicitly stated that his particular interpretation of 'Ricardo's problem' suggested itself as a natural consequence of the development of the ideas of the Standard system and the distinction of basic from non-basic goods that emerged in the course of the investigation of his book. The significance of the Standard commodity was so central to Sraffa's theoretical analysis that when Manara (1980, [1968] came up with a few examples of joint production cases where apparently no Standard commodity in the real space could exist, Sraffa's reaction was most revealing:

> One might conclude that the [possibility to construct] a Standard product [is under the same restrictive conditions] as any other general theory in *an interdependent industries system*. If this is true, M. would have destroyed my humble hut and as a new Samson, he would have torn down the entire palace of modern economics. ~~che Adam Smith in~~ … If the examples [given by M.] were verified, together with my humble building all the milestones of traditional economics would be swept away.
>
> (Sraffa n.d., D3/14: 61 and D3/14: 63, large brackets in original, emphasis added)[1]

[1] See Verger (2021) for Sraffa's last unpublished paper; and for a critique and a response see Schefold (2021) and Sinha and Verger (2021). Also see Dupertuis and Sinha (2009b) for our solution to Manara Problem.

Now P & T and their associates should at least rethink their position on the significance of Sraffa's Standard commodity for his theory. Furthermore, P & T also do not realize that $\boldsymbol{w}$ is only a measure of wages in terms of the Standard commodity and it was a significant theoretical move for Sraffa to have switched from taking the wages as given from outside the system of price equations to taking the rate of profits as given from outside once the above given equation was established through the aid of the Standard system and the Standard commodity (Sraffa 1960, 33). As we have shown above, within the classical perspective, no commodity residue and therefore, no Standard system exists. In this framework, even on the assumption of the condition of equal rate of profits, the average rate of profits can be known only if either prices are known or are determined simultaneously with it. Since in this framework, profits do not take an independent meaning till prices are determined, the rate of profits cannot be taken as given from outside the system of equations independently of the knowledge of prices. Therefore, the only option for Garegnani and his followers is to take real wages as given from outside. But Sraffa, with the help of the Standard system and the Standard commodity could establish the average rate of profits without the knowledge of prices and therefore was able to break from the classical tradition of taking real wages as given from outside the system of equations. Now, the advantage of taking the rate of profits from outside is that it immediately reveals the measure of wages in terms of a ratio of the net Standard output without even having the knowledge of what constitutes the Standard commodity let alone having to conceive wages to be consisting of the Standard commodity (see Sinha 2022b for a discussion on this point).

This should clarify that there is no conceptual difference in determining the rate of profits and the maximum rate of profits once the wages measured in terms of the Standard commodity is removed from the system and that's why Sraffa showed no objection to Manara's examples of the joint-production cases that also assumes zero wages (see Verger 2021 and Sinha & Verger 2021).

Moreover, as we have shown above, Sraffa's proof that there always exists *one and only one* Standard system associated with any system of basic goods was crucial to his analysis as well as ours. But P & T claim that 'Clearly we can find *three* Standard systems'! (P & T, Appendix, p. 180).

Since I have already placed my arguments before the reader, I see no reason to take up their repetitive arguments in any detail except to point out that they seem to have a habit of characterising their misunderstandings as my 'error'. They accuse me of contradicting myself or make

an error when I suggest that the average rate of profits of the Standard system is independent of prices and since the average rate of profits of the observed system must be equal to the average of the Standard system, it constrains the prices to a unique set that is associated with all industries receiving the average rate of profits – in other words, it is the average rate of profits that constrains the prices to a unique set and not the other way around. I, however, see no contradiction or error in this. For P & T 'market prices' have to be independent of the average rate of profits as they live in the world of independent industries and therefore, they consistently misinterpret my statement that the average rate of profits is determined independently of the knowledge of prices as if it amounts to stating that in the observed system the average rate of profits will remain constant irrespective of what prices are applied to the system. '… that Sinha has to fall back to what is probably the most perplexing aspect of his general position: namely, the idea according to which, as is the case with the average rate of profit of the Standard system, the average rate of profit of the real economic system too would be independent of prices' (P & T, p. 177). In the light of my arguments presented above, the reader can easily verify that I do not make any such claim. All I claim is that in an interconnected system of industries that is not in the Standard proportion prices have no freedom – there is only one set of prices that is admissible in the system, which is associated with the average rate of profits of the system, which must be equal to the average rate of profits of its Standard counterpart and that is determined independently of prices – which is exactly what Sraffa also means when he writes, 'The same rate of profits, which in the Standard system is obtained as a ratio between *quantities* of commodities, will in the actual system result from the ratio of aggregate *values*.' (Sraffa 1960, 22-23).

P & T also misunderstand my expression that 'the maximum rate of expansion of the two systems must be equal' to mean a reference to the 'maximum rate of growth of the system', which, of course, I do not mean, since in Sraffa there is no presumption about how the surplus is used in the next time period. What I mean by expansion is that a surplus has been produced, which is a physical property of the productive system and its physical measure is given by R*, which must be the same as R in the observed system as the two systems of productions are the same only organized in different proportions. But the fact that the physical system logically cannot grow at a rate higher than R* should have made P & T to pause and think, why this physical measure must triumph over their psychological measure of the average rate of profits when it comes to the maximum growth potential of the system? What my critics do not comprehend is the idea of an interconnected unitary system since they come from a classical world where all industries are independent.

Where Do Stefano Perri and Gianmarco Oro Go Wrong?

One of the fundamental properties of Sraffa's system of basic goods is that if one industry is deleted then the whole system dies – *i.e.*, the system can no longer be reduced. Therefore, one cannot find a solution of the system without taking the whole system into account. P & O, nevertheless, derive from equations of Sraffa's basic goods system the conclusion: 'We immediately see that it is possible to divide this system into two parts: the first two equations constitute a determined system with two unknowns. Thus, we know the prices of iron and coal in terms of wheat, and the third equation can only determine the deviation from the general rate of profit and thus the value of the sectoral rate of profit as a dependent variable' (P & O, p. 197). The fundamental mistake P & O make throughout their paper is that though they are dealing with a system of interconnected basic goods, they treat all industries as independent from each other (as P & T do) and therefore happily assign arbitrary values to either prices or industrial rates of profits. They state, 'The standard system is built to define the measure of prices that is invariable to income distribution changes' (P & O, p. 200). As a matter of fact, it is not the Standard system but rather the Standard commodity that is used as the unit of measure for prices and wages but more importantly, the Standard commodity remains unaffected by price changes due to changes in r, when the condition of a uniform industrial rate of profits holds and not in the cases where arbitrary prices are imposed on the system with unequal industrial rates of profits, as is the case with their examples.

In the appendix to their paper, P & O apply three positive and negative deviations from the average rate of profits to the Standard system and show that its average remains constant at 20% to prove that I'm wrong to assert that all deviations from the average rate of profits in the Actual system must be zero (P & O, eq. 6, p. 203). They don't seem to understand that it is universally accepted, and it is clear as daylight, that the average rate of profits of the Standard system is independent of prices, so no matter what prices are applied in these equations it will not affect the average. My proposition is about the Actual system and it holds only if the average rate of profits of the Actual system is equal to the average rate of profits of the Standard system but P & O's arbitrary values of rates of profits give 19% as the average rate of profits of the Actual system and not 20%. So how could it prove my proposition wrong?

My critics are a close-knit group who cite each other's criticisms of my interpretation of Sraffa with adjectives such as 'remarkable', 'authoritative' 'persuasive', etc. without ever even mentioning my responses to

those criticisms let alone engaging with them. I suspect this response will suffer the same fate. So I leave it to the open minded reader to judge for him or herself as to which side the stronger argument and evidence lie?

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Appendix
Why Profit Rates 'Must' be Uniform in Sraffa's system[1]

An overly informal argument that the profit rate in Sraffa's system 'must' be uniform was made in the Appendix to Sinha (2016).[2] Misunderstandings about what we are arguing appear to derive in part from differing conceptions of the meaning of the word 'must'. Confusion could have been avoided by making a more formal statement, as is done here. As we clarify, the claim being made is that only uniform profit rates ensure a systemic property insisted upon by Sraffa: that the average profit rate of a standard system is the same as the average profit rate of any 'actual' system associated with it.

1. *Definitions*

Define a 'Sraffian system of industrial equations' (SSIE) for a set of basic goods[3] as a set of accounting relationships among these goods, which we may refer to as industrial equations, relating physical input and physical output quantities observed in each industry (industries identified by $i \in \{1, \ldots, n\}$, possibly unknown prices ($\boldsymbol{p}$, a vector, whose components are the prices of the goods produced by each industry, i) at which they are valued, and possibly unknown profit rates in each industry ($\boldsymbol{R}$, a vector, with components R_i corresponding to individual industrial profit rates). For ease of exposition, we work with the case in which wages are set to zero and profit rates are resultingly maximal, but nothing hinges on this simplification. The profit rate of an industry, i, is defined as the total value of outputs minus the total value of inputs in a given industry, expressed as a share of the total value of inputs in that industry. The physical inputs in each industry are denoted by $\boldsymbol{a}_i$ a vector (making up together a set of such vectors $\{\boldsymbol{a}_i\}$) and the physical output by b_i a scalar (*i.e.* the outputs taken together forming an output vector, $\boldsymbol{b}$). An 'industrial equation' from the SSEI is given by (where $\circ$ represents the inner product):

[1] Sanjay G. Reddy and Ajit Sinha.
[2] Ajit Sinha, *A Revolution in Economic Theory: The Economics of Pierro Sraffa*, Palgrave Macmillan/Springer.
[3] As defined by Sraffa, a basic good is a good that, directly or indirectly, enters as an input into the production of all goods.

$$(\boldsymbol{p} \circ \boldsymbol{a}_i)\,(1 + R_i) = \boldsymbol{p}_i \circ b_i\,.$$

Define a rescaled industrial equation as one in which the inputs and outputs have been multiplied on both sides by the same scalar factor (which may be called the rescaling multiplier). Further, define a rescaled SSIE as one in which one or more of the industrial equations have been rescaled.

Note that for a given industrial equation *all* of the inputs and outputs have to be multiplied by the same value for the resulting change to count as a rescaling. [If a multiplier is applied for an industry but some of the resulting change in the value of the inputs is then 'redistributed' to or from the profit rate (for instance by treating more output as having been produced from the same inputs, at a newly higher profit rate) then this does not count as a rescaling in the sense that we have employed the term]. Therefore, if we define a 'solution' to an SSIE as a set of prices, and the associated profit rates, $\{\boldsymbol{p}, \boldsymbol{R}\}$ that satisfies its equations, rescaling an SSIE will lead to a new SSIE according to the previous definition, but the solution will remain unchanged.

Define a 'uniform profit rate solution' to an SSIE as one in which all profit rates are equal and a 'non-uniform profit rate solution' to an SSIE as one in which they are not all equal.

Define the 'average profit rate' of an SSIE as the total value of outputs of the economy minus the total value of inputs of the economy, expressed as a share of the total value of inputs of the economy (where the 'economy' is defined as the set of industries in the SSIE, and 'value' is calculated at prices, $\boldsymbol{p}$), *i.e.* the average profit rate is $\overline{R} = \dfrac{\boldsymbol{p} \circ (\boldsymbol{b} - \boldsymbol{a})}{\boldsymbol{p} \circ \boldsymbol{a}}$.

Define the 'value of output identity' as the requirement that the average profit rate must be the same when calculating the output in two different ways, viz. by valuing the output as the sum of the values, at given prices, of industry-specific inputs marked up by the industry-specific profit rates, and by valuing the output as the sum of the values, at the same prices, of industry-specific outputs. There is a value of output identity for each industry, which when summed provides such an identity for the economy as a whole:

$$\frac{\sum_{i=1}^{n} (\boldsymbol{a}_i \circ \boldsymbol{p})\,(1 + R_i)}{\sum_{i=1}^{n} (\boldsymbol{a}_i \circ \boldsymbol{p})} - 1 = \frac{\boldsymbol{b} \circ \boldsymbol{p}}{\sum_{i=1}^{n} (\boldsymbol{a}_i \circ \boldsymbol{p})} - 1\,.$$

II. *The Profit Rate as a Systemic Property*

The average profit rate is an important property of a system of basic goods which can be related to another important such property, the

productivity of the standard system.[1] The application of a systemic perspective, eschewing reductionism and embracing holism, was a central methodological precept for Sraffa, as argued in Sinha (2016). This perspective draws our attention to the following proposition.

Consider an SSIE corresponding to an original or 'actual'[2] system, X, which is not a standard system, *i.e.* is not a system for which the output vector of the economy is a multiple of its input vector. Assume that X constitutes a full rank system,[3] so that prices are fully determined given the profit rate(s). It has an associated standard system, S(X) related to X by rescaling.

Proposition:
Only the uniform profit rate solution for X possesses the property that the calculated average profit rate in X is necessarily the same as the productivity of the associated standard system, S(X).

Proof:
We first show that the uniform profit rate solution possesses the property and then show that a non-uniform profit rate solution does not. Consider any solution for X, call it $\zeta^\star = \{\boldsymbol{p}^\star, \boldsymbol{R}^\star\}$ which may or may not involve uniform profit rates. The input cost in an industry is $\boldsymbol{a}_i \circ \boldsymbol{p}^\star$, and the industry-specific input value marked up by the industry specific profit rate is $(\boldsymbol{a}_i \circ \boldsymbol{p}^\star)(1 + R_i)$. Since $\zeta^\star$ is a solution, by definition $(\boldsymbol{a}_i \circ \boldsymbol{p}^\star)(1 + R_i) = b_i p_i^\star$. That is to say, the industry specific output value is the same as the industry-specific input value marked up by the industry-specific profit rate, for each and every industry.

Summing the last statement up across industries, $\Sigma_{i=1}^n (\boldsymbol{a}_i \circ \boldsymbol{p}^\star)(1 + R_i) = \Sigma_{i=1}^n b_i p_i^\star$. As expected, dividing both sides by the total input costs $\Sigma_{i=1}^n (\boldsymbol{a}_i \circ \boldsymbol{p}^\star)$ and subtracting unity from each side to arrive at the average profit rate tells us that the two ways of calculating it give identical results – the value of output identity for the economy: $\frac{\Sigma_{i=1}^n (\boldsymbol{a}_i \circ \boldsymbol{p}^*)(1+R_i)}{\Sigma_{i=1}^n (\boldsymbol{a}_i \circ \boldsymbol{p}^*)} - 1 = \frac{\Sigma_{i=1}^n b_i p_i^*}{\Sigma_{i=1}^n (\boldsymbol{a}_i \circ \boldsymbol{p}^*)} - 1$. When the profit rates are identical the left-hand side simplifies to the uniform profit rate, R.

What about the standard system, S(X)? In order to arrive at S(X) we must rescale one or more industrial equations by multipliers. Call the multipliers that are applied (which are not identical since X is not itself

[1] It should be noted that only a system of basic goods has a unique standard system associated with it.

[2] Sraffa's term.

[3] This is not a challenging assumption, since it amounts to presuming that each of the industrial equations is needed to provide a full description of the input and output relationships of the economy, and cannot be deduced from the others.

a standard system) m_i. By rescaling the value of output identity for each industry by its multiplier and adding the resulting expressions we arrive at:

$$\sum_{i=1}^{n} m_i (\boldsymbol{a}_i \circ \boldsymbol{p}^*)(1+R_i) = \sum_{i=1}^{n} m_i b_i p_i^* .$$

Note that the average profit rate as calculated for the standard system necessarily equals the physical productivity, π, of the standard system, S(X), since

$$\frac{\sum_{i=1}^{n} m_i (\boldsymbol{a}_i \circ \boldsymbol{p}^*)(1+Ri)}{\sum_{i=1}^{n} m_i (\boldsymbol{a}_i \circ \boldsymbol{p}^*)} - 1 = \frac{\sum_{i=1}^{n} m_i b_i p_i^*}{\sum_{i=1}^{n} m_i (\boldsymbol{a}_i \circ \boldsymbol{p}^*)} - 1 .$$

And the RHS of this expression necessarily equals π since, for all possible prices, the ratio of the value of the economy's outputs to the value of the economy's inputs is, by definition of the standard system, $\pi + 1$:

$$\left(\frac{\sum_{i=1}^{n} m_i b_i p_i^*}{\sum_{i=1}^{n} m_i (\boldsymbol{a}_i \circ \boldsymbol{p}^*)} - 1 \right) = \pi .$$

Therefore, $\dfrac{\sum_{i=1}^{n} m_i (\boldsymbol{a}_i \circ \boldsymbol{p}^*)(1+R_i)}{\sum_{i=1}^{n} m_i (\boldsymbol{a}_i \circ \boldsymbol{p}^*)} - 1 = \pi .$

When is the calculated average profit rate of the original system the same as the productivity of the associated standard system?

The calculated average profit rate of the original system is

$$\frac{\sum_{i=1}^{n} (\boldsymbol{a}_i \circ \boldsymbol{p}^*)(1+R_i)}{\sum_{i=1}^{n} (\boldsymbol{a}_i \circ \boldsymbol{p}^*)} - 1 .$$

As noted that when profit rates are uniform this calculated average profit rate reduces to R, the common profit rate. The same is also true of the immediately prior expression: for the average profit rate of the standard system. Therefore, when profit rates are uniform, the average profit rate of the original system is equal to the productivity of the associated standard system.

What if profit rates are non-uniform? Since under the hypothesis the calculated profit rate is the same for both the original system and the associated standard system, therefore

$$\frac{\sum_{i=1}^{n} m_i (\boldsymbol{a}_i \circ \boldsymbol{p}^*)(1+R_i)}{\sum_{i=1}^{n} m_i (\boldsymbol{a}_i \circ \boldsymbol{p}^*)} - 1 = \frac{\sum_{i=1}^{n} (\boldsymbol{a}_i \circ \boldsymbol{p}^*)(1+R_i)}{\sum_{i=1}^{n} (\boldsymbol{a}_i \circ \boldsymbol{p}^*)} - 1 .$$

Which implies that (introducing new indices to avoid ambiguity):

$$\sum_{j=1}^{n} (\boldsymbol{a}_j \circ \boldsymbol{p}^*) \sum_{i=1}^{n} (m_i)(\boldsymbol{a}_i \circ \boldsymbol{p}^*)(1+R_i) = \sum_{j=1}^{n} m_j (\boldsymbol{a}_j \circ \boldsymbol{p}^*) \sum_{i=1}^{n} (\boldsymbol{a}_i \circ \boldsymbol{p}^*)(1+R_i) .$$

Rearranging:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(\boldsymbol{a}_j \circ \boldsymbol{p}^{\star})(m_i)(\boldsymbol{a}_i \circ \boldsymbol{p}^{\star})(1+R_i) = \sum_{i=1}^{n}\sum_{j=1}^{n}(\boldsymbol{a}_j \circ \boldsymbol{p}^{\star})(m_j)(\boldsymbol{a}_i \circ \boldsymbol{p}^{\star})(1+R_i).$$

It is easy to see that this expression is satisfied for the uniform profit rate case, regardless of the rescaling factors, $\{m_i\}$, but is there any other case for which it is satisfied? Rearranging further we arrive at a final expression, which we call A*:

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(m_i - m_j)(\boldsymbol{a}_j \circ \boldsymbol{p}^{\star})(\boldsymbol{a}_i \circ \boldsymbol{p}^{\star})(1+R_i) = 0.$$

Clearly, A* can be satisfied if $m_j = m_i$ for all i,j, but in this case the original system *is* a standard system, which we have presumed is not the case.

Is there any *other* case in which the expression is satisfied? Note that the multipliers $\{m_i\}$ are functions of the inputs and outputs $\{\boldsymbol{a}_i\}$ and $\boldsymbol{b}$ that define the SSIE.[1] Recall that given a set of profit rates $\{R_i\}$ the SSIE fully determines $\boldsymbol{p}^{\star}$. Thus, by varying $\{R_i\}$, $\boldsymbol{p}^{\star}$ would be forced to vary, without there being any change in $\{m_i\}$ nor in $\{\boldsymbol{a}_i\}$.[2] Note too that statement A* is a non-linear expression in $\mathbf{p}^{\star}$ which cannot be constructed as a linear combination of the industrial equations in the SSIE. It can be confirmed that A* is an additional requirement that is being imposed in the presence of an already fully determined solution – and therefore is not always satisfied.[3] If there *ever* exists a non-uniform profit rate solution for an SSIE that also satisfies A* (a question which we do not further explore here) it would appear to arise only in a highly specific case.[4]

It follows that the calculated average profit rate in the original system is *necessarily* the same as the productivity of the associated standard system in only one case: uniform profit rates. QED.

The Proposition provides the sense in which it was argued that the profit rate 'must' be uniform. This sense of necessity follows from the

[1] More specifically, the ratios in which they stand to each other are functions of the inputs and outputs. But since the multipliers are given by the components of an eigenvector associated with the matrix of inputs and outputs, there is no explicit formula for this dependence in higher dimensions, as is well known. Specifically, due to the Abel-Ruffini theorem there can be no such formula in the case where the number of industries is greater than four.

[2] When the profit rates are (or the uniform profit rate is) given in Sraffa's system of interconnected basic goods, then the prices are determined simultaneously by the resulting set of linear equations. On the other hand, if all of the prices are taken as given, then each industry equation suffices to independently determine the profit rate for each individual industry.

[3] Specifically, it is possible to construct non-uniform profit rate solutions of the original system that have a calculated average profit rate different from the productivity of the associated standard system. It is therefore clear that a solution of the original system does not automatically satisfy the requirement.

[4] Indeed, it seems a reasonable inference that such a solution would 'almost always' not exist, but a rigorous proof would need to be provided that the set of such solutions, if it is non-empty, is of measure zero. We leave the question of whether any such solutions exist and how they may be characterized unexplored.

significance of holistic properties in Sraffa's vision of a system of basic goods – and in particular from the importance of the average profit rate of an actual system being identical to the productivity of the associated standard system.[1]

[1] Joseph Schumpeter's concept of a 'vision' underpinning any economic theory is apposite here.

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